Coordinates
Time and Space

Location of Objects On and Off the Earth

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Time

- Universal Time (UT)
- Local Civil Time (LCT)
- International Atomic Time (TAI)
- Coordinated Universal Time (UTC)
- Global Positioning System Time (GPS)
- Long Range Navigation Time (Loran-C)
- Sidereal Time (ST)
- Ephemeris Time (ET)
- Terrestrial Dynamic Time (TDT)
- Barycentric Dynamic Time (TDB)
- Amateur astronomers assume UT = UTC = GMT

Universal Time (UT)

- Related to the motion of the Sun at the Greenwich meridian (longitude 0).
- At longitude 0 the Sun reaches its maximum altitude at 12:00 PM
- The Sun is observed to return to the same position in the sky after exactly 24 hours have elapsed.
- UT varies with the irregular rotation of the Earth.
  - The 9.1 magnitude Sumatra earthquake in 2004 shortened the day by 6.8 microseconds.
  - The 8.8 magnitude Chile earthquake in 2010 shortened the day by 1.26 microseconds.
- British Summer Time (BST) is used during the summer to fit the working day more conveniently into the daylight hours in Great Britain. BST = UT + 1
- Daylight Savings Time (DST) is the name for the adjusted time in other parts of the world.

Local Civil Time (LCT)

- Local Civil Time is the time we use every day. This time is Universal Time adjusted for the time zone and when appropriate for daylight savings time.
  - For no Daylight Savings Time
    $LCT = UT + TimeZoneOffset$
  - For Daylight Savings Time
    $LCT = UT + TimeZoneOffset + 1$
- The Sun is observed to return to the same position in the sky after exactly 24 hours have elapsed.

International Atomic Time (TAI)

- International Atomic Time (TAI) is the scale of time resulting from analysis by the Bureau International de l'Heure in Paris using atomic standards in many countries.

Coordinated Universal Time (UTC)

- The basis of legal time keeping on Earth
- Derived from TAI in such a manner as to be within 0.9 seconds of UT and a whole number of seconds different from TAI.
  - On July 21, 1985, TAI – UTC = 23 seconds.
  - On August 7, 2020, TAI – UTC = 37 seconds.
- This relationship is achieved by including occasional leap seconds in UTC, usually at the end of June or December.
- UTC = TAI – (number of leap seconds)
- $UT - 0.9 < UTC < UT + 0.9$

Global Positioning Time (GPS)

- Global Positioning System time, is the atomic time scale implemented by the atomic clocks in the GPS ground control stations and in the GPS satellites themselves.
- GPS time was zero at 0 hour, January 6, 1980
- GPS time is not perturbed by leap seconds.
- On August 7, 2020, GPS – UTC = 18 seconds.

Long Range Navigation Time (Loran-C)

- Long Range Navigation time is an atomic time scale implemented by the atomic clocks in Loran-C chain transmitter sites.
- Loran-C time was zero at 0 hour, January 1, 1958
- Loran-C time is not perturbed by leap seconds.
- On August 7, 2020, Loran-C – UTC = 27 seconds.

Sidereal Time (ST)

- Time based on a clock whose rate is such that any star, other than the Sun, is observed to return to the same position in the sky after exactly 24 hours have elapsed.
- Sidereal Time is not the same as solar time, of which UT is an example, because during the course of one solar day the Earth moves nearly 1 degree along its orbit around the Sun.
- There are 365¼ solar days in a year. During this time, the Earth makes 366¼ revolutions about its own axis; therefore, there are 366¼ sidereal days in a year.
- Each sidereal day is slightly shorter that a solar day.
- Sidereal Time = Hour Angle (H) of the Vernal Equinox.

Ephemeris Time (ET)

- Adopted for use by astronomers before 1984
- Astronomers need a system of time which is uniform since the theories of celestial mechanics assume that such a quantity exists.
- Calculated from the motion of the moon and assumed to be uniform (not uniform enough for the accuracies needed in today’s measurements).

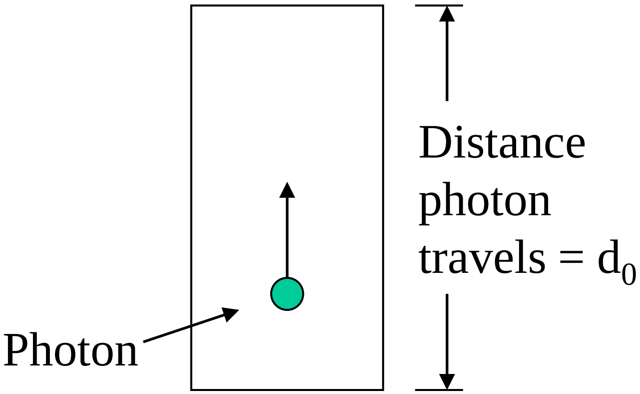
Terrestrial Dynamic Time (TDT)

- A system of time based upon atomic time, not effected by the irregular rotation of the Earth about its axis.
- TDT replaced ET at the beginning of 1984
- TDT = TAI + 32.184 seconds

Barycentric Dynamic Time (TDB)

- Currently used by astronomers to calculate planetary motions.
- TDB is the same as TDT except for relativistic corrections to move the origin of the solar system barycenter (center of mass of a system of bodies).
- Corrections to TDT are periodic with a magnitude of 1.6 milliseconds and an average value of zero.
- TDB = TDT + 0.001658 sin(g) + 0.000014 sin(2g)
  - g = 357.53 + 0.9856003 (JD – 2451545.0)
  - JD (Julian date) is simply a continuous count of days and fractions of days since noon Universal Time on January 1, 4713 BCE (on the Julian calendar).
- TDT – 0.0016 < TDB < TDT + 0.0016

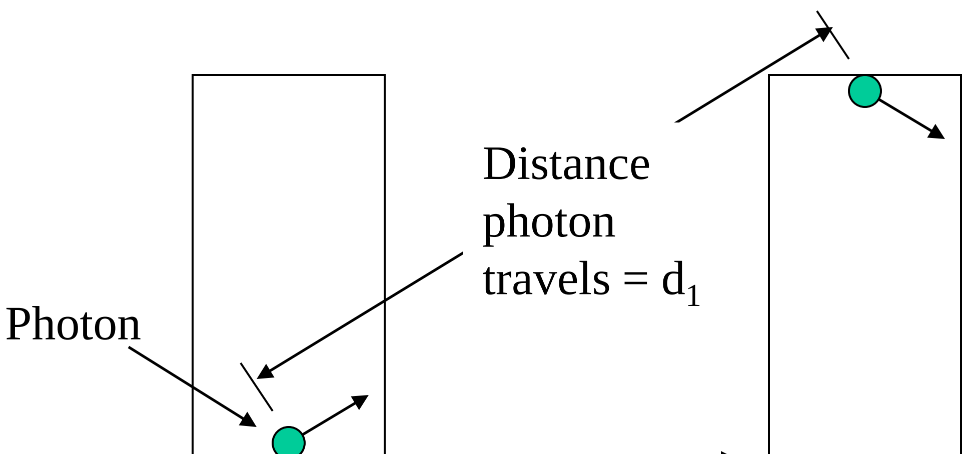
Time on a Moving Reference



Stationary Light Clock

Each tic of the clock occurs when the photon bounces off the end of the clock.
A unit of time t_u passes when the photon travels distance d_0.
$d_0 = c * t_u$ or $t_u = d_0/c$, where c is the speed of light

A unit of time on the moving reference passes when the photon travels distance d_0 as seen from that reference. $t_u = t_0 = d_0/c$



As seen from a stationary reference
$t_u = \mathrm{sqrt}(d_1^2 - d_{clk}^2)/c$
$t_u = \mathrm{sqrt}(c^2 * t_0^2 - v_{clk}^2 * t_0^2)/c$
$t_u = t_0 * \mathrm{sqrt}(1 - v_{clk}^2/c^2)$

Time passes slower as v_{clk} approaches the speed of light c.

Moving Light Clock (distance moved = $d_{clk} = v_{clk} * t_0$)

Time on an Accelerating Reference

On the accelerating reference $d_0 = c * t_u$ or $t_u = t_0 = d_0/c$

From a stationary reference $t_u = sqrt(d_1^2 – d_{clk}^2)/c$

$t_u = sqrt(c^2 * t_0^2 – d_{clk}^2)/c$

where $d_{clk} = v_0 * t_0 + a_0 * t_0^2/2$

$t_u = sqrt(c^2 * t_0^2 – (v_0 + a_0 * t_0/2)^2 * t_0^2)/c$

$t_u = t_0 * sqrt(1 – (v_0 + a_0 * t_0/2)^2/c^2)$

If $v_0 = 0$ then $t_u = t_0 * sqrt(1 – a_0^2/c^2 * t_0^2/4)$

and if $a_0 = g$ then $t_u = t_0 * sqrt(1 – g^2/c^2 * t_0^2/4)$

Time passes slower for an object in very high acceleration or in a very strong gravity field.

Coordinate Systems used for Locating Objects off the Planet

- Horizon Coordinates
- Equatorial Coordinates
- Ecliptic Coordinates
- Galactic Coordinates

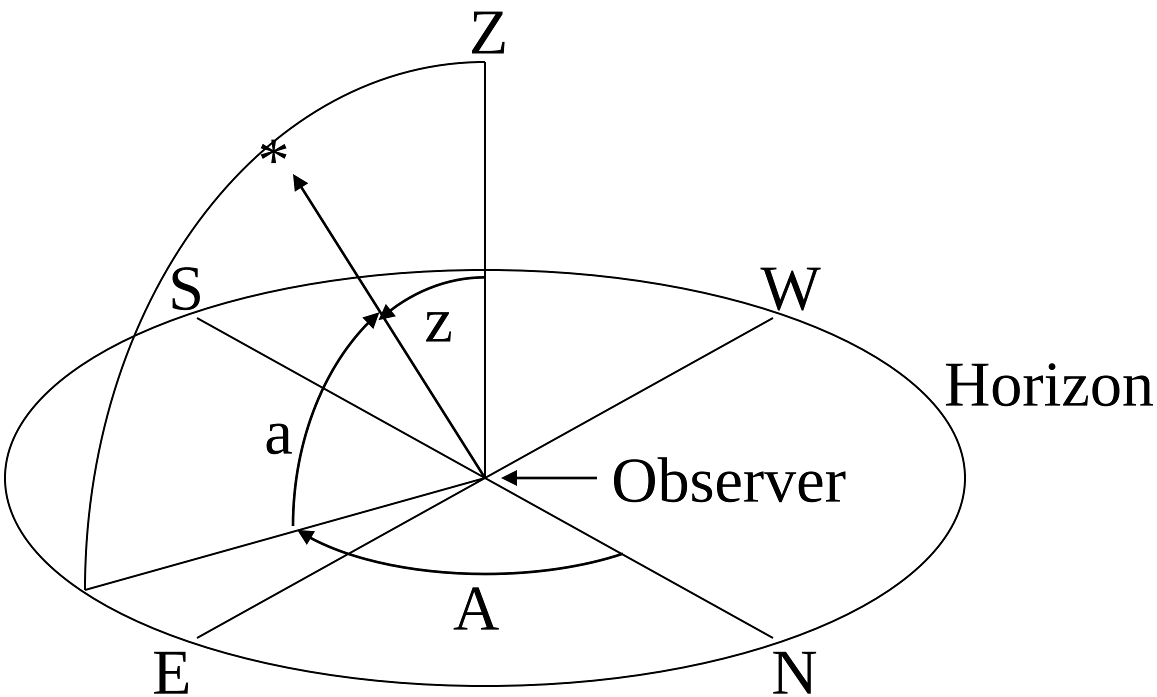
Horizon Coordinates

- Altitude (a)
  The angle a ray from the observer to the object makes above the plane of the observer’s horizon (measured in degrees in an upward direction).
- Azimuth (A)
  The angle the projection of the above ray on the plane of the observer’s horizon makes from the observer’s true north direction (measured in degrees in a clockwise direction).

Horizon Coordinates (Cont.)

- Zenith (z)
  The angle a ray, from the observer to the object, makes with the vertical (measured in degrees in a downward direction).
  - Zenith can be used in place of Altitude
  - $z = 90 – a$

Horizon Coordinates (Cont.)



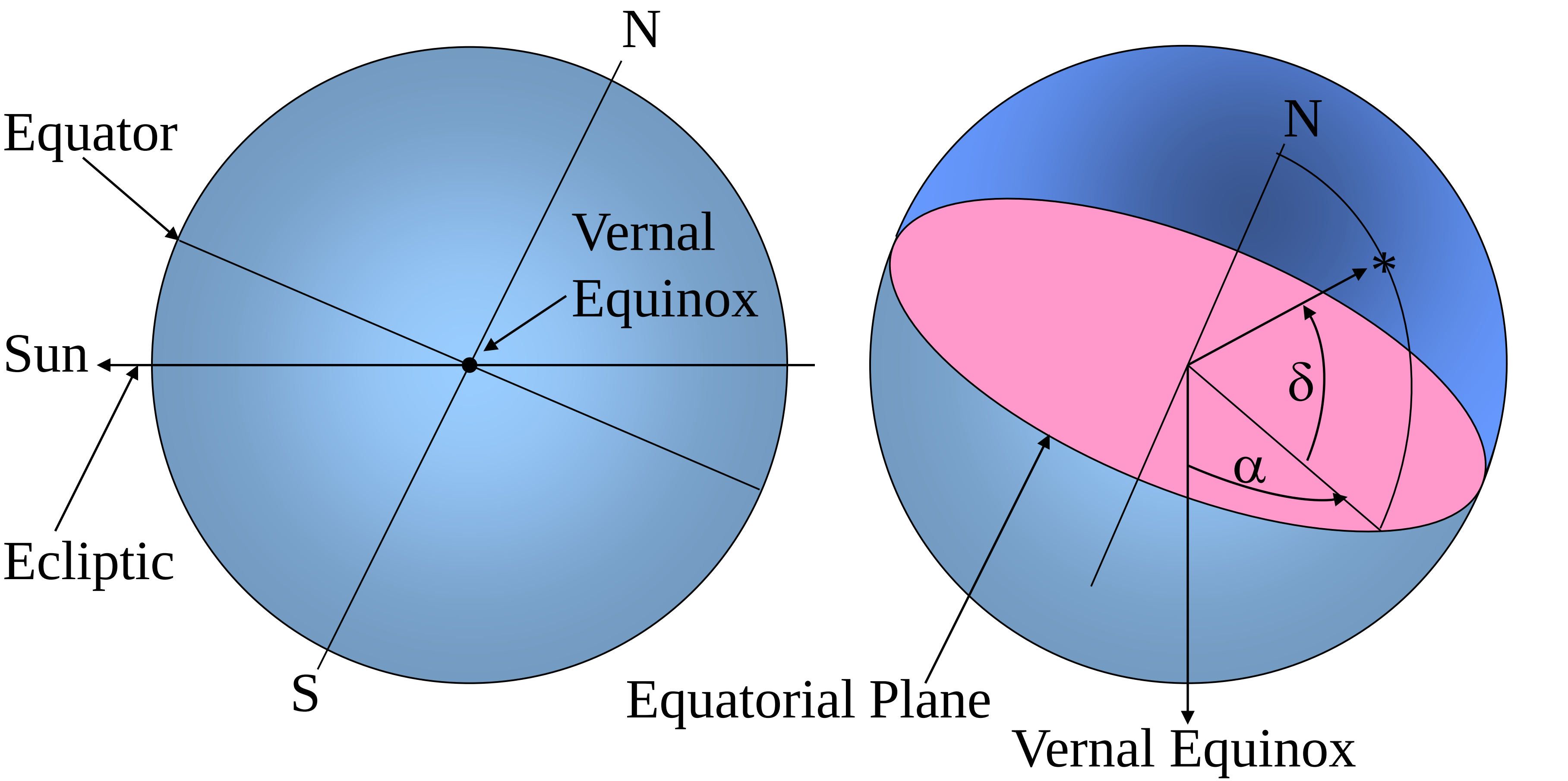
Equatorial Coordinates

- Declination (δ)
  The angle a ray, from the center of the Earth to the object, makes above the equatorial plane (measured in degrees in a northerly direction).
- Right Ascension (α)
  The angle the projection of the above ray on the equatorial plane makes from the Vernal Equinox which is also know as the First Point of Aries (measured in degrees in an easterly direction).

Equatorial Coordinates (Cont.)

- Vernal Equinox (γ)
  The direction that lies along the line of the intersection of the plane of the Earth's equator with that of the Earth's orbit around the sun.
  - By definition this is the direction toward the Sun on March 21st, the Spring Equinox.
  - Also referred to as the First Point of Aries even though it points to the constellation Pisces.
- Hour Angle (H)
  $H = LST - \alpha$

Equatorial Coordinates (Cont.)



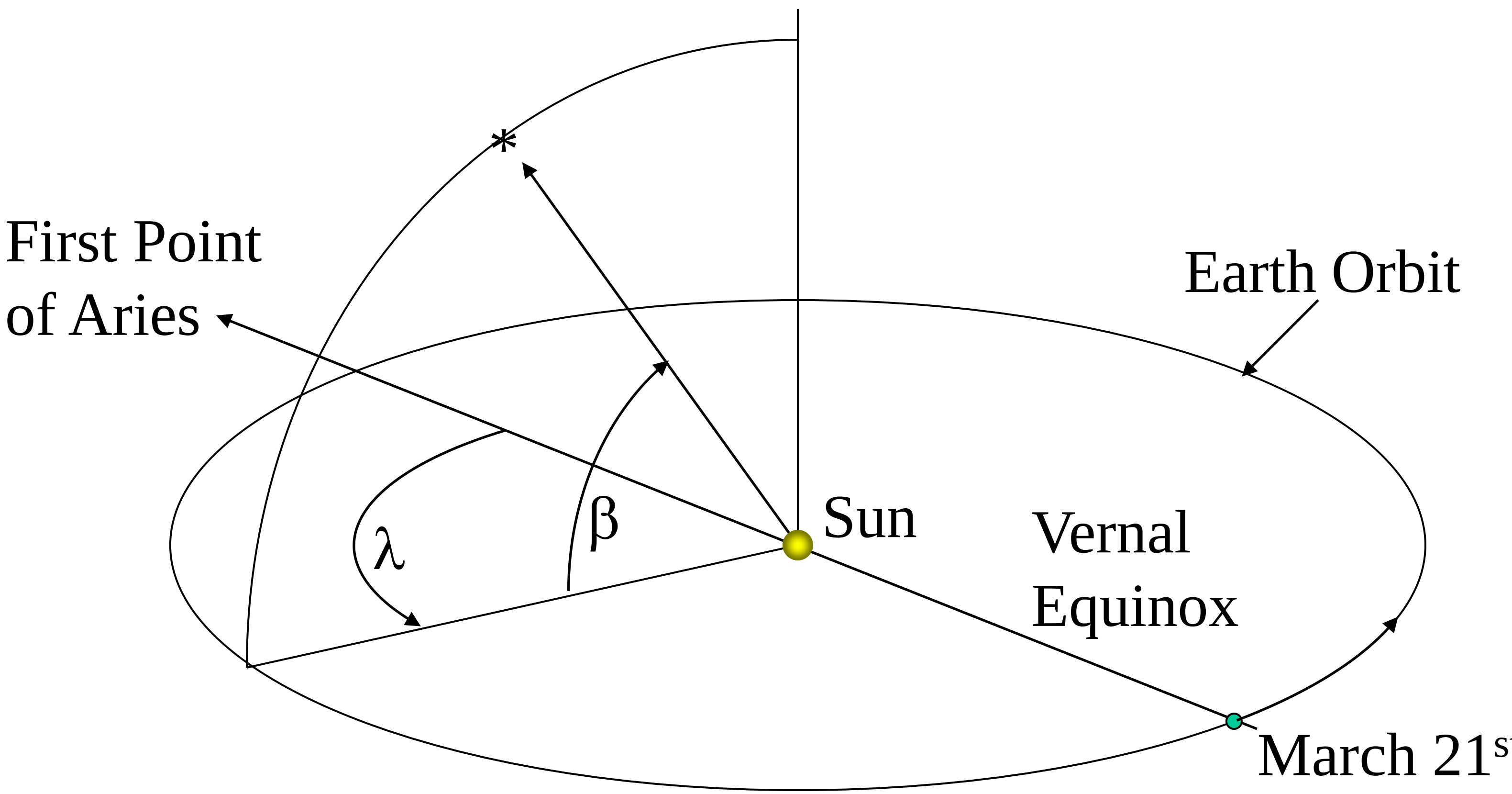
Horizon ⬄ Equatorial

- Horizon to Equatorial
  $\sin(\delta) = \sin(a)\sin(\phi) + \cos(a)\cos(\phi)\cos(A)$
  $\cos(H) = [\sin(a) - \sin(\phi)\sin(\delta)]/[\cos(\phi)\cos(\delta)]$
- Equatorial to Horizon
  $\sin(a) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(H)$
  $\cos(A) = [\sin(\delta) - \sin(\phi)\sin(a)]/[\cos(\phi)\cos(a)]$

Ecliptic Coordinates

- The Ecliptic is the plane containing the Earth's orbit around the Sun.
- Ecliptic Latitude (β)
  The angle a ray from the Sun to the object makes above the Ecliptic.
- Ecliptic Longitude (λ)
  The angle the projection of the above ray on the Ecliptic makes from the vernal equinox measured in an easterly direction.

Ecliptic Coordinates (Cont.)



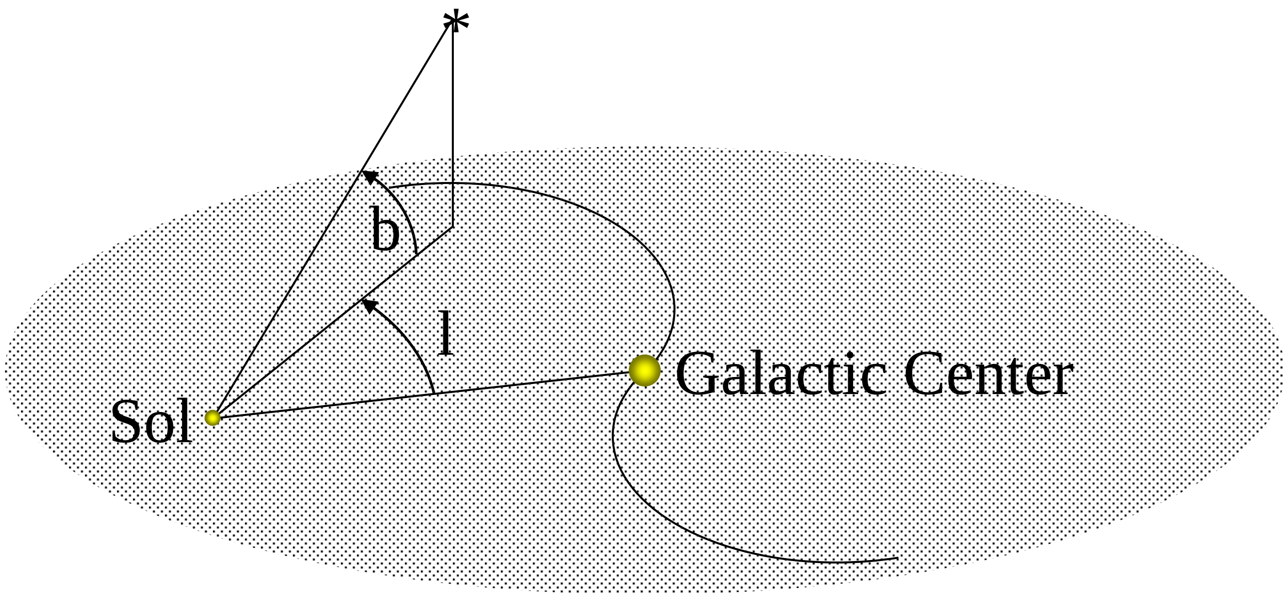
Equatorial $\Leftrightarrow$ Ecliptic

- Equatorial to Ecliptic
  $\lambda = \tan^{-1}\{[\sin(\alpha)\cos(\varepsilon) + \tan(\delta)\sin(\varepsilon)]/\cos(\alpha)\}$
  $\beta = \sin^{-1}\{\sin(\delta)\cos(\varepsilon) – \cos(\delta)\sin(\varepsilon)\sin(\alpha)\}$
- Ecliptic to Equatorial
  $\alpha = \tan^{-1}\{[\sin(\lambda)\cos(\varepsilon) – \tan(\beta)\sin(\varepsilon)]/\cos(\lambda)\}$
  $\delta = \sin^{-1}\{\sin(\beta)\cos(\varepsilon) + \cos(\beta)\sin(\varepsilon)\sin(\lambda)\}$
- ε is the obliquity of the ecliptic, the angle between the planes of the equator and the ecliptic.
  $\varepsilon = 23.441884$ degrees

Galactic Coordinates

- Galactic Latitude (b)
  The angle a ray from the Sun to the object makes above the plane of the Galaxy.
- Galactic Longitude (l)
  The angle the projection of the above ray on the Galactic plane makes from the direction of the center of the Galaxy. This angle increases in the same direction as right ascension.

Galactic Coordinates (Cont.)



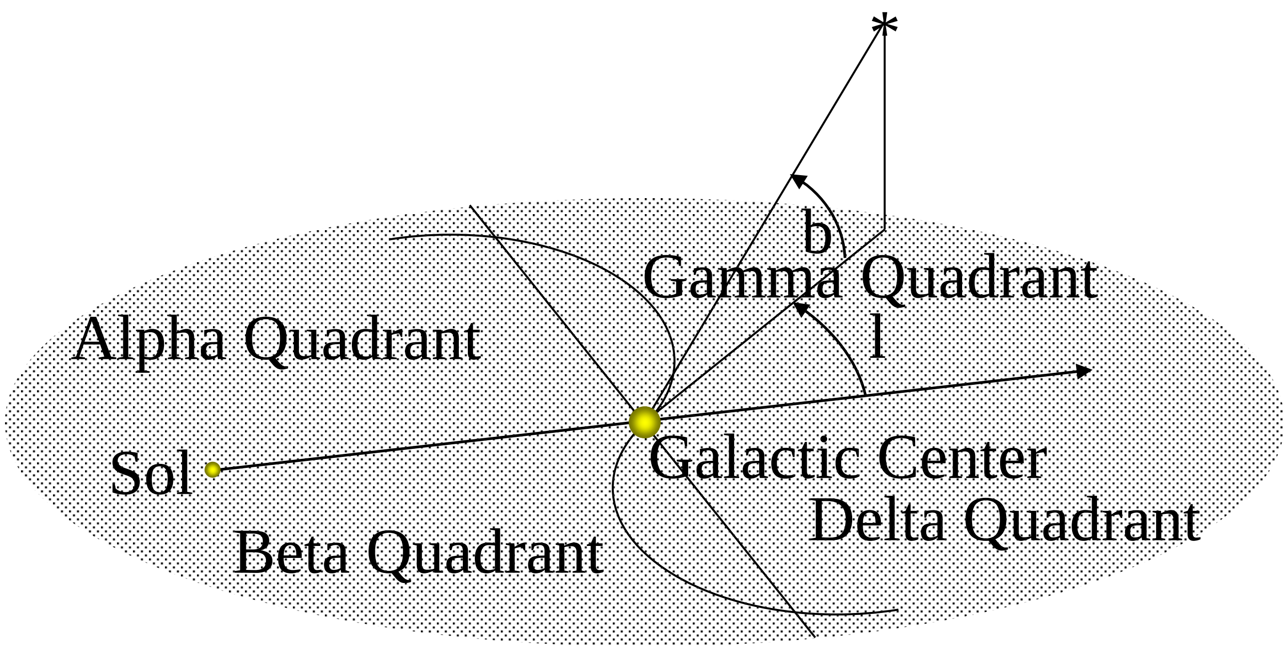
Equatorial ⇔ Galactic

- Equatorial to Galactic
  $b = \sin^{-1}\{\cos(\delta)\cos(27.4)\cos(\alpha - 192.25) + \sin(\delta)\sin(27.4)\}$
  $l = \tan^{-1}\{[\sin(\delta) - \sin(b)\sin(27.4)]/[\cos(\delta)\sin(\alpha - 192.25)\cos(27.4)]\} + 33$
- Galactic to Equatorial
  $\delta = \sin^{-1}\{\cos(b)\cos(27.4)\sin(l - 33) + \sin(b)\sin(l - 33)\}$
  $\alpha = \tan^{-1}\{[\cos(b)\cos(l - 33)]/[\sin(b)\cos(27.4) - \cos(b)\sin(27.4)\sin(l - 33)]\} + 192.5$

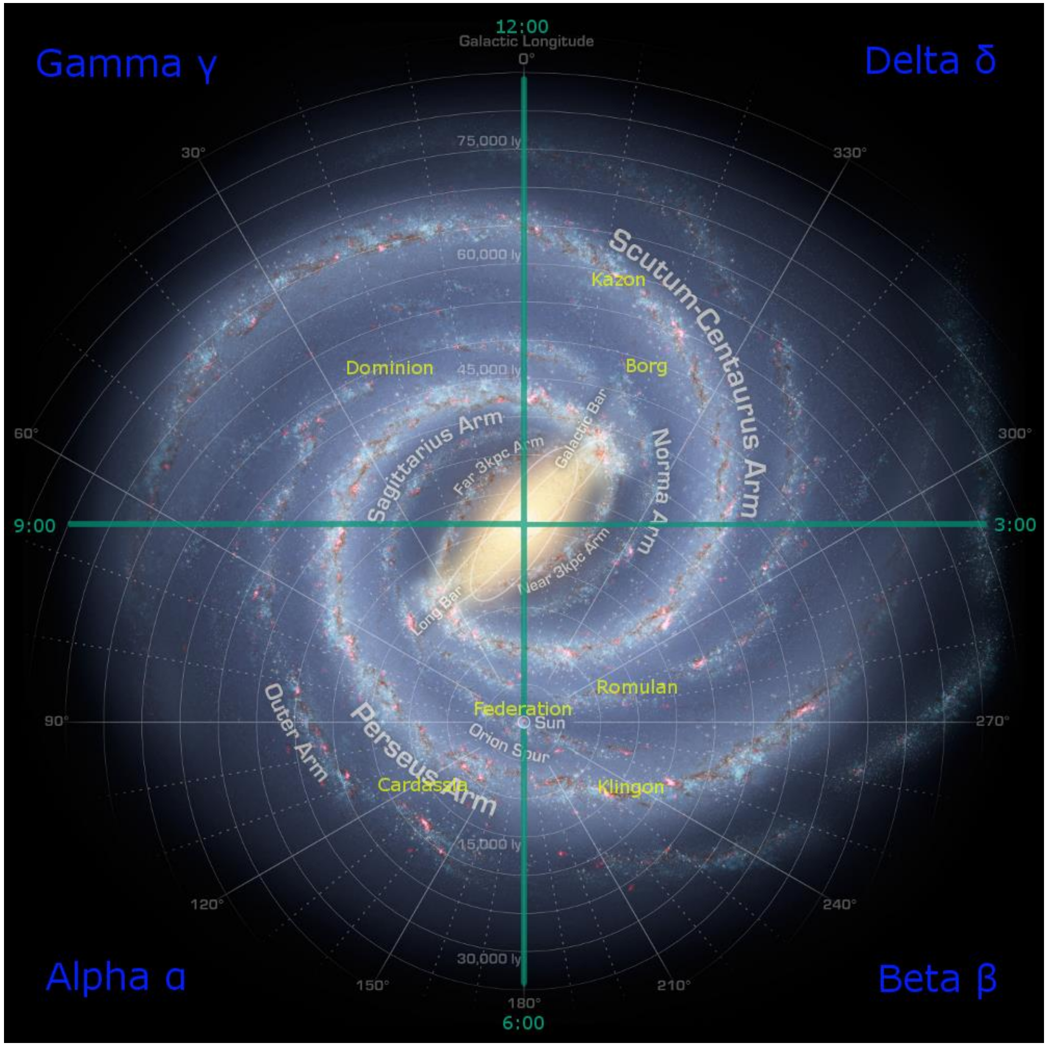
Star Trek Galactic Coordinates (fictional)

- Galactic Latitude (b)
  The angle a ray, from the center of the Galaxy to the object, makes above the plane of the Galaxy.
- Galactic Longitude (l)
  The angle the projection of the above ray on the Galactic plane makes from the direction of a ray from the Sun through the Galactic Center. This angle increases in the same direction as right ascension.

Star Trek Galactic Coordinates (Cont.)



Star Trek Galaxy Map



Coordinate Systems used for Locating Objects on the Planet

- Latitude, Longitude and Altitude
- Universal Transverse Mercator (UTM)
- Military Grid Reference System (MGRS)
- Latitude, Longitude, Altitude relative to mean sea level ellipsoid (spheroid)
- Earth Fixed Geocentric or Earth Fixed Greenwich (EFG)
- Inertial EFG

The Problem with Latitude and Longitude

- The shape of the Earth is an irregular geoid.
- The shape is approximated by a Spheroid (commonly referred to as an ellipsoid).
  - Mathematical simplification.
  - The center of the geoid does not coincide with the center of the spheroid .
- Looking from either pole the shape of the Earth is approximated by a circle.
- Looking from the equator the shape of the Earth approximated by an ellipse.
- Spheroidal coordinates are related to measured coordinates by datums.
  - Various datums are used to minimize the disagreement between spheroidal and measured coordinates at different locations on the Earth.
  - WGS-84 datum is used by most GPS receivers in the United States
    - Military Real Time Differential GPS Accuracy = 2 to 7 meters
    - Military Accuracy under Selective Availability (SA) and Anti-Spoofing (A/S) = 21 meters (latitude disagreement < 0.01’).
    - Commercial Accuracy with SA Engaged = 100 meters (latitude disagreement < 0.05’).

What is Latitude?

- Terrestrial Latitude (λ_t): The angle a ray makes from the center of the Earth geoid to the celestial equatorial plane.
- Geocentric Latitude (λ_c): The angle a ray makes from the center of the standard ellipsoid to the equatorial plane of the standard ellipsoid.
- Astronomic Latitude (λ_a): The angle a normal to the geoid makes to the celestial equatorial plane.
- Geodetic Latitude (λ): The angle a normal to the standard ellipsoid makes to the equatorial plane of the standard ellipsoid.

Determining Latitude

- Determination made using position of North Star (Polaris)
- Determination made using calendar and sun
  - Solar latitude obtained from calendar
  - Latitude = mid day zenith angle + solar latitude
- Determination made using a GPS receiver.
  - GPS receiver calculates Geodetic Latitude.
  - Geodetic Latitude converted to Astronomic Latitude via WGS-84 Datum (lookup table).

What is Longitude?

- Terrestrial Longitude (ϕ_t): The angle the projection of a ray on the equatorial plane makes from the center of the Earth geoid to the Greenwich meridian.
- Geocentric Longitude (ϕ_c): The angle the projection of a ray on the equatorial plane makes from the center of the standard ellipsoid to the Greenwich meridian.
- Astronomic Longitude (ϕ_a): The angle the projection of a normal to the geoid on the equatorial plane makes to the Greenwich meridian.
- Geodetic Longitude (ϕ): The angle the projection of a normal to the standard ellipsoid on the equatorial plane makes to the Greenwich meridian.

Determining Longitude

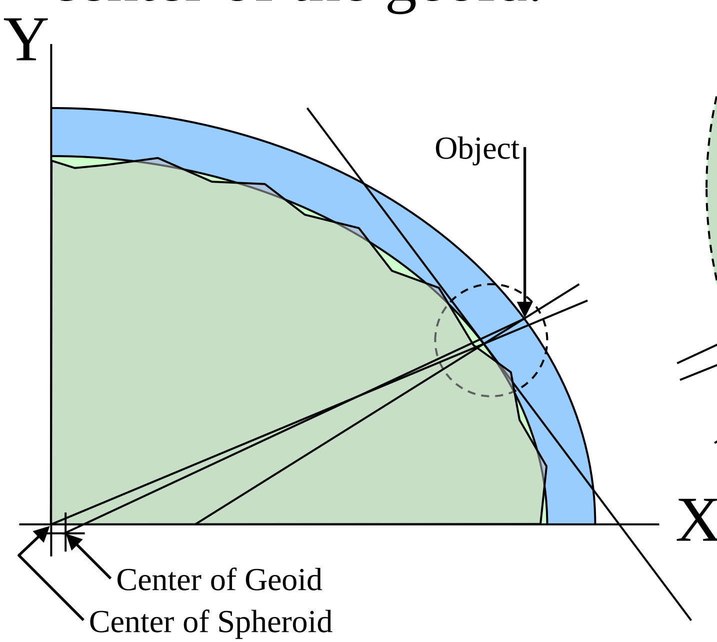
- Determination made using astronomic measurements.
  - Position of planets and stars
  - Position of sun
- Determination made using time relative to Greenwich meridian and position of the sun.
- Determination made using a GPS receiver.
  - GPS receiver calculates Geodetic Longitude.
  - Geodetic Longitude converted to Astronomic Longitude via WGS-84 Datum (lookup table).

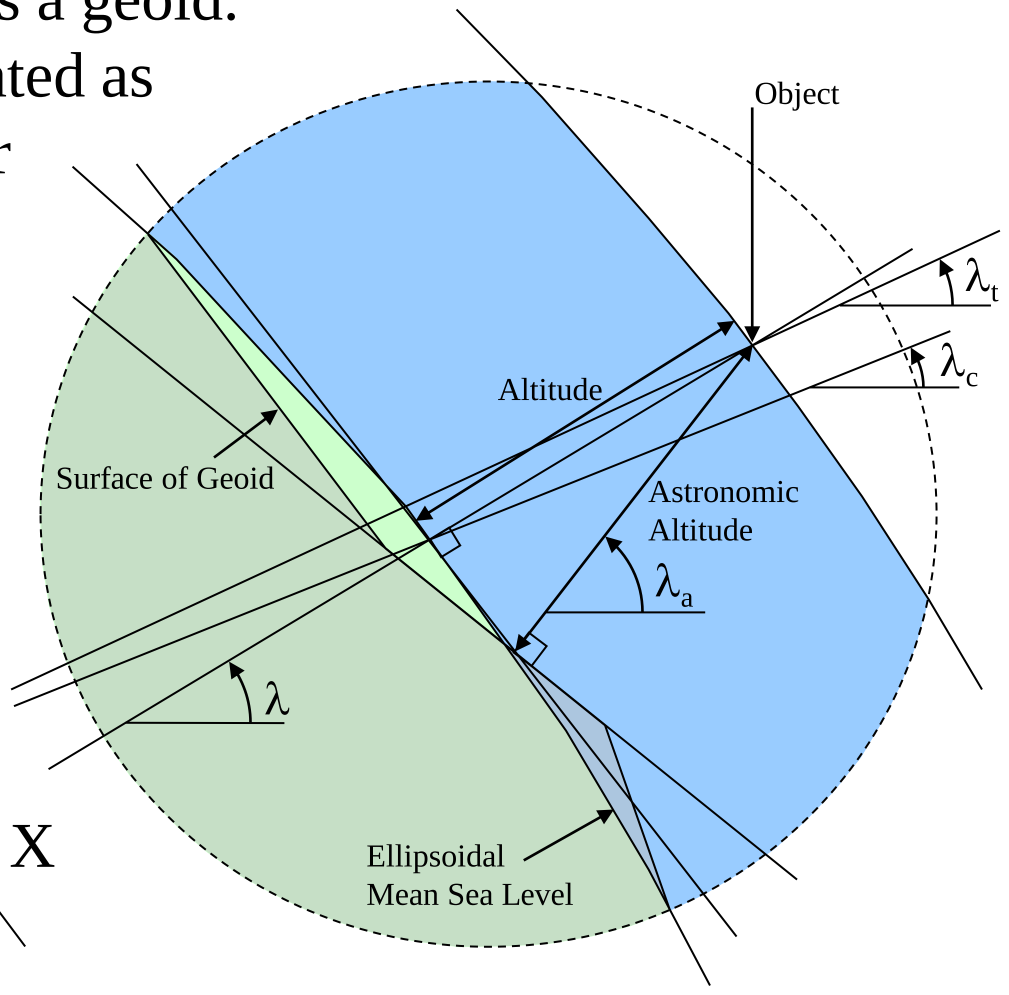
Determining Direction

- From the North Star (Polaris)
- From the rising and setting of the Sun or Moon
- From the crescent of the Moon
- From a Sun Compass
- From a magnetic compass (allowing for magnetic declination)
- From a GPS receiver

The Shape of the Earth

The shape of the Earth is a geoid. This shape is approximated as a Spheroid whose center is not located at the center of the geoid.





Coordinate Transformation Approximate Shape of Earth

- Spheroid

$x^2 + (R_e/R_p)^2 * y^2 = R_e^2$ and $x = (R_e/R_p) * (R_p^2 - y^2)^{1/2}$

$\tan(\lambda_c) = y / x$

$x = R_e/(1 + (R_e/R_p)^2 * \tan^2(\lambda_c))^{1/2}$, $y = x * \tan(\lambda_c)$

$\tan(\lambda) = -dx/dy = (R_e/R_p) * y/(R_p^2 - y^2)^{1/2} = (R_e/R_p)^2 * \tan(\lambda_c)$

$x = R_e/(1 + (R_p/R_e)^2 * \tan^2(\lambda))^{1/2}$, $y = x * (R_p/R_e)^2 * \tan(\lambda)$

where:

R_e = Radius at Spheroid's Equator

R_p = Radius at Spheroid's Pole

λ_c = Geocentric Latitude

λ = Geodetic Latitude

Coordinate Transformation Geodetic to Geocentric Cartesian

- Given Geodetic Coordinates (λ, ϕ, A)
  Calculate Mean Sea Level Geocentric Coordinates (E_o , F_o , G_o)

  $E_o = R_e/(1 + (R_p/R_e)^2 * \tan^2(\lambda))^{1/2} * \cos(\phi)$

  $F_o = R_e/(1 + (R_p/R_e)^2 * \tan^2(\lambda))^{1/2} * \sin(\phi)$

  $G_o = R_e/(1 + (R_p/R_e)^2 * \tan^2(\lambda))^{1/2} * (R_p/R_e)^2 * \tan(\lambda)$

  Or substituting $(R_p/R_e)^2 = 1 – e^2$ where ‘e’ is the eccentricity

  $E_o = R_e * \cos(\lambda)/(1 – e^2 * \sin^2(\lambda))^{1/2} * \cos(\phi)$

  $F_o = R_e * \cos(\lambda)/(1 – e^2 * \sin^2(\lambda))^{1/2} * \sin(\phi)$

  $G_o = R_e * \cos(\lambda)/(1 – e^2 * \sin^2(\lambda))^{1/2} * (1 – e^2) * \tan(\lambda)$

- Given Altitude Calculate Object Geocentric Coordinates

  $E_A = E_o + A * \cos(\lambda) * \cos(\phi)$

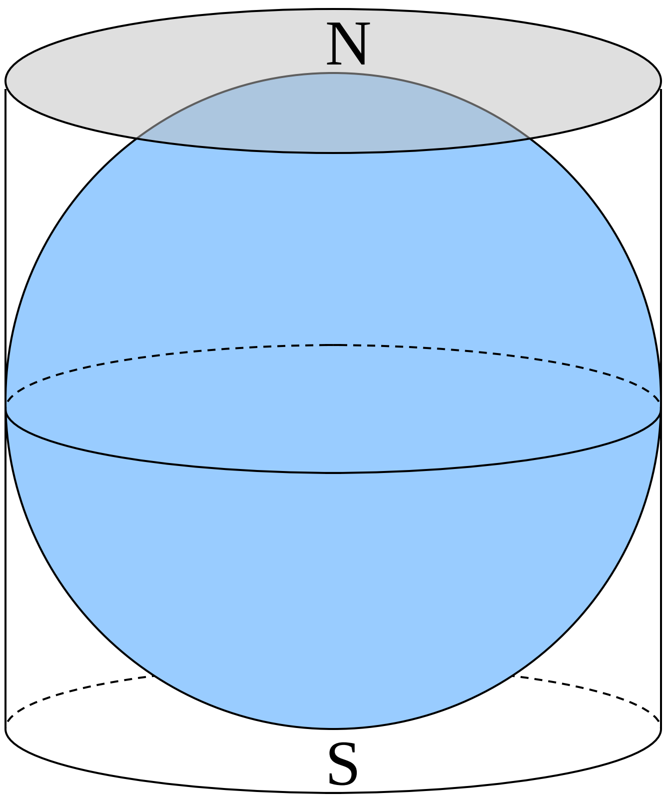
  $F_A = F_o + A * \cos(\lambda) * \sin(\phi)$

  $G_A = G_o + A * \sin(\lambda)$

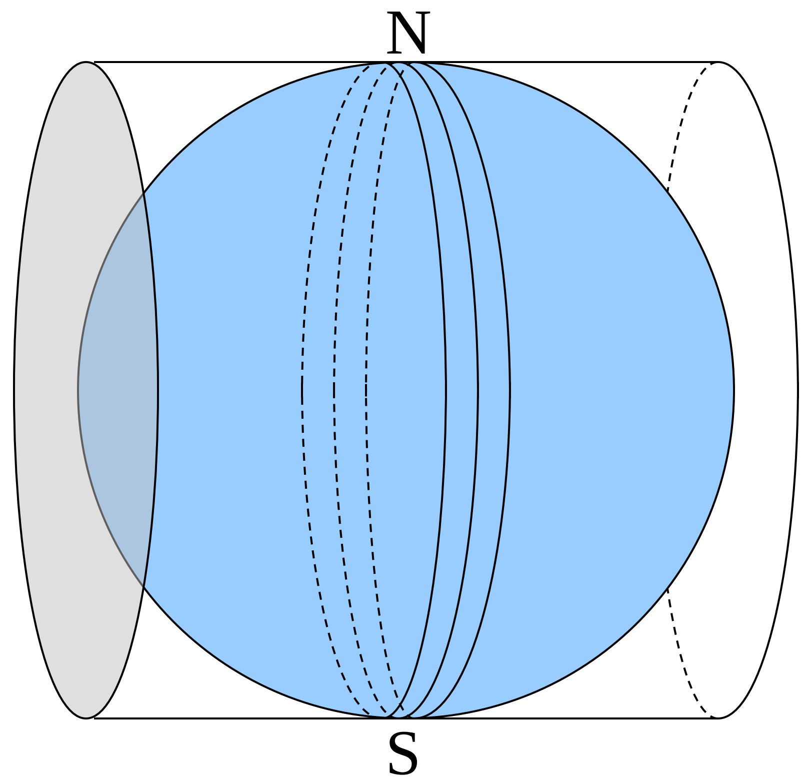
Geocentric to Geodetic

- Given Object Geocentric Coordinates (E_A, F_A , G_A)
  Calculate Geodetic Coordinates (λ, ϕ, A)
  - Longitude
    if($E_A > 0$) $\phi = \text{atan}(F_A / E_A)$
    else if($E_A < 0$) $\phi = \text{atan}(F_A / E_A) + 180$
    else if($F_A > 0$) $\phi = 90$
    else $\phi = -90$
  - Latitude and Altitude
    Let $X_A = (E_A * E_A + F_A * F_A)^{1/2}$ and $X_0 = R_e/(1 + (R_p/R_e)^2 * \tan^2(\lambda))^{1/2}$
    $G_A = X_A * ((R_p+A)/(R_e+A))^2 * \tan(\lambda)$, $G_0 = X_0 * (R_p/R_e)^2 * \tan(\lambda)$
    $\lambda = \text{atan}(G_A/(X_A * ((R_p+A)/(R_e+A))^2)) = \text{atan}(G_0/(X_0 * (R_p/R_e)^2))$
    $A = ((G_A - G_0)^2 + (X_A - X_0)^2)^{1/2}$
    First calculate λ assuming A = 0
    Next calculate A from given G_A and X_A and calculated G_0 and X_0 resulting from λ
    Re-calculate λ from G_A, X_A and A
    Compare new and old λ and repeat above two steps until difference is within allowed value
    Set allowed difference to $1 * 10^{-15}$ radians

Mercator Projections



Mercator Projection

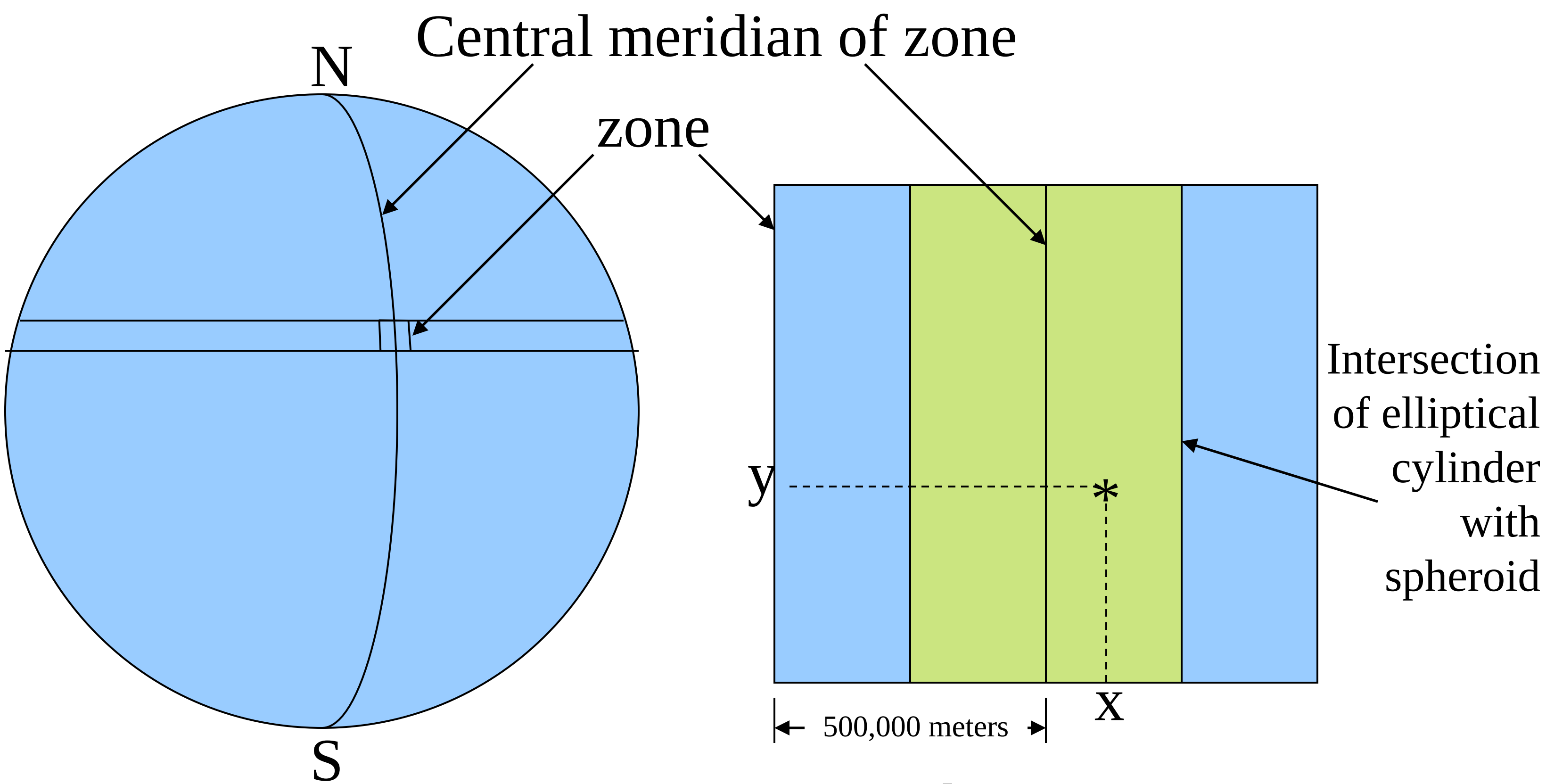


Transverse Mercator Projection

UTM Description

- An artificial grid system dividing the globe longitudinally into strips 6 degrees wide and divided latitudinally mostly 8 degrees tall.
- The grids are numbered in an easterly direction from 1 to 60 starting at -180 degrees longitude.
- The grids are lettered (not always used) in a northerly direction from A to Z starting at the South Pole (I and O are omitted).
  - South of 80 degrees south it is divided into two zones A and B
  - From 80 degrees south to 72 degrees north into 19 zones C through W
  - From 72 degrees north to 84 degrees north zone X
  - North of 84 degrees north it is divided into two zones Y and Z
- Each zone is further divided decimally
  - From the Central Meridian with a 500 kilometer (km) false easting to ensure a positive number.
  - When lettered latitudinal zones are not used the zone is numbered from the Equator (0 km) or to the Equator (10,000 km) in a northerly direction.
- The Polar Stereographic projection is usually used in the polar regions.

Universal Transverse Mercator (UTM)

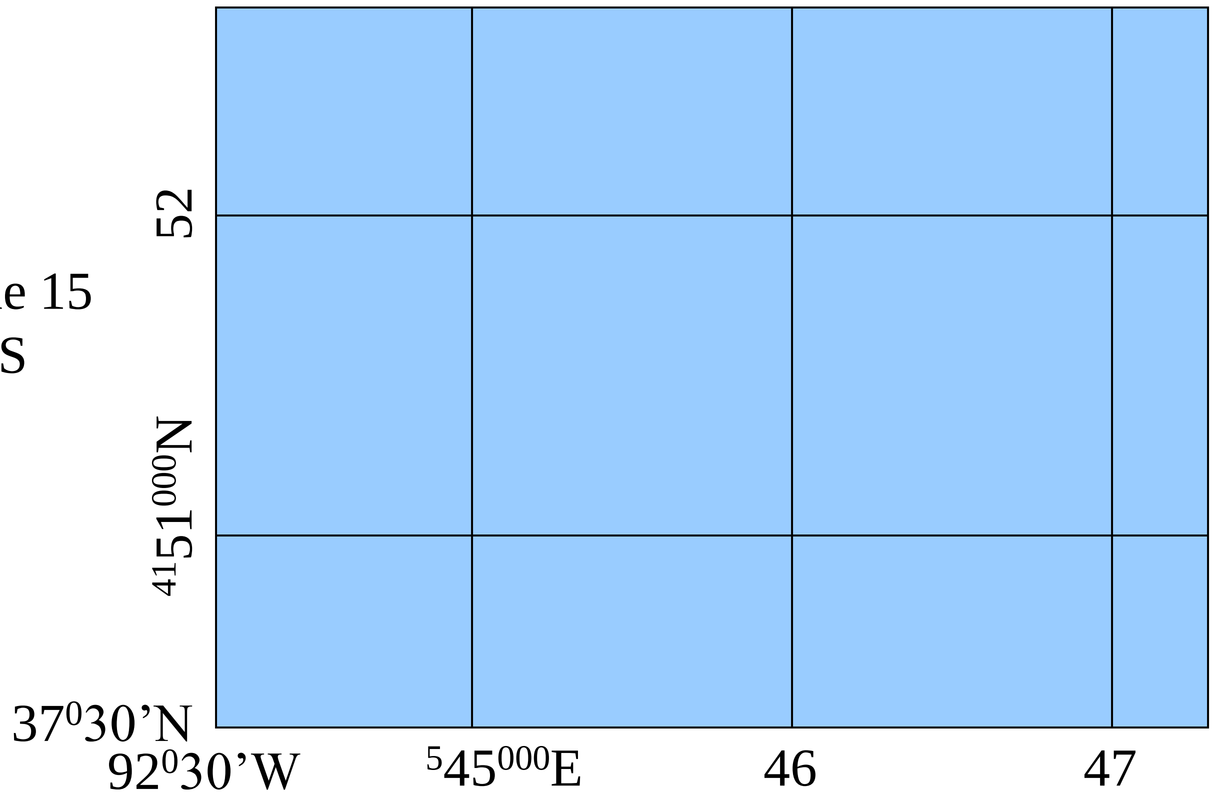


y = northing, x = easting

Reading a Map in UTM

Easting is always a 6-digit coordinate
Northing is always a 7-digit coordinate

Longitude Zone 15
Latitude Zone S



WGS-84 Parameters

- Equatorial Radius of Earth (a)
  a = 6,378,137 meters
- Polar Radius of Earth (b)
  b = 6,356,752.3142 meters
- Flattening ((a-b)/a)
  (a-b)/a = 1/298.257223563

Symbols Used in Conversion to UTM

- lat: latitude of location
- lon: longitude of location
- lon_0: central meridian of zone
- k_0: scale along lon_0 = 0.9996
- e = $SQRT(1-b^2/a^2)$ = .08 approximately. This is the eccentricity of the earth's elliptical cross-section.
- $e'^2 = (ea/b)^2 = e^2/(1-e^2)$ = .007 approximately.
- n = (a-b)/(a+b)
- rho = $a(1-e^2)/(1-e^2\sin^2(lat))$: Radius of curvature of Earth in the meridian plane
- nu = $a/(1-e^2\sin^2(lat))^{1/2}$
  - This is the radius of curvature of the earth perpendicular to the meridian plane.
  - It is also the distance from the point in question to the polar axis, measured perpendicular to the earth's surface.
- p = $(lon-lon_0)$
- sin1'' = sine of one second of arc = pi/(180*60*60) = 4.8481368×10^{-6}

Calculate the Meridional Arc

- Meridional Arc (S)
  - $S = A'lat - B'\sin(2lat) + C'\sin(4lat) - D'\sin(6lat) + E'\sin(8lat)$
    - $A' = a[1 - n + (5/4)(n^2 - n^3) + (81/64)(n^4 - n^5) ...]$
    - $B' = (3an/2)[1 - n + (7/8)(n^2 - n^3) + (55/64)(n^4 - n^5) ...]$
    - $C' = (15an^2/16)[1 - n + (3/4)(n^2 - n^3) \; ...]$
    - $D' = (35an^3/48)[1 - n + (11/16)(n^2 - n^3) \; ...]$
    - $E' = (315an^4/51)[1 - n \; ...]$
  - lat is in radians
- The USGS gives this form, which may be more appealing to some. (They use M where the Army uses S)
  - $M = a[(1 - e^2/4 - 3e^4/64 - 5e^6/256 ....)lat$
    $- (3e^2/8 + 3e^4/32 + 45e^6/1024...)\sin(2lat)$
    $+ (15e^4/256 + 45e^6/1024 + ...)\sin(4lat)$
    $- (35e^6/3072 + ...) \sin(6lat) + ...)]$
  - lat is in radians

Latitude Longitude to UTM

- UTM Zone
  - Longitude Zone (1 through 60)
    Zone = (lon + 180)/6 rounded up
  - Latitude Zone (A through Z omitting I and O)
    Zone = A and B from 90S to 80S
    Zone = C from 80S to 72S
    Zone = D, E, …, W every 8 degree north
    Zone = X from 72N to 84N
    Zone = Y and Z from 84N to 90N

Latitude Longitude to UTM

- UTM northing and easting
  - northing = y = $K1 + K2p^2 + K3p^4$
    - $K1 = Sk_0$
    - $K2 = k_0 \sin^2 1''$ nu sin(lat)cos(lat)/2
    - $K3 = [k_0 \sin^4 1''$ nu $\sin(lat)\cos^3(lat)/24][(5 - \tan^2(lat) + 9e'^2\cos^2(lat) + 4e'^4\cos^4(lat)]$
  - easting = x = $K4p + K5p^3$
    - $K4 = k_0 \sin 1''$ nu cos(lat)
    - $K5 = (k_0 \sin^3 1''$ nu $\cos^3(lat)/6)[1 - \tan^2(lat) + e'^2\cos^2(lat)]$
    - Easting x is relative to the central meridian. For conventional UTM easting add 500,000 meters to x.

Parameters used in Conversion to Latitude Longitude

- northing = y, easting = x (relative to central meridian) Subtract 500,000 from conventional UTM coordinate.
- Calculate the Meridional Arc (M)
  $M = y/k_0$
- Calculate Footprint Latitude (fp)
  fp = mu + J1sin(2mu) + J2sin(4mu) + J3sin(6mu) + J4sin(8mu)
  - $mu = M/[a(1 - e^2/4 - 3e^4/64 - 5e^6/256...)$
  - $e_1 = [1 - (1 - e^2)^{1/2}]/[1 + (1 - e^2)^{1/2}]$
  - $J1 = (3e_1/2 - 27e_1^3/32 ..)$
  - $J2 = (21e_1^2/16 - 55e_1^4/32 ..)$
  - $J3 = (151e_1^3/96 ..)$
  - $J4 = (1097e_1^4/512 ..)$

Symbols used in Conversion to Latitude Longitude

- $e'^2 = (ea/b)^2 = e^2/(1 - e^2)$
- $C1 = e'^2\cos^2(fp)$
- $T1 = \tan^2(fp)$
- $R1 = a(1 - e^2)/(1 - e^2\sin^2(fp))^{3/2}$
  This is the same as rho in the forward conversion formulas above, but calculated for fp instead of lat.
- $N1 = a/(1 - e^2\sin^2(fp))^{1/2}$
  This is the same as nu in the forward conversion formulas above, but calculated for fp instead of lat.
- $D = x/(N1k_0)$

UTM to Latitude Longitude

- UTM to Latitude Longitude
  - lat = fp - Q1(Q2 - Q3 + Q4)
    - Q1 = N1 tan(fp)/R1
    - Q2 = $(D^2/2)$
    - Q3 = $(5 + 3T1 + 10C1 - 4C1^2 - 9e'^2)D^4/24$
    - Q4 = $(61 + 90T1 + 298C1 + 45T1^2 - 3C1^2 - 252e'^2)D^6/720$
  - lon = lon_0 + (Q5 - Q6 + Q7)/cos(fp)
    - Q5 = D
    - Q6 = $(1 + 2T1 + C1)D^3/6$
    - Q7 = $(5 - 2C1 + 28T1 - 3C1^2 + 8e'^2 + 24T1^2)D^5/120$

Military Grid Reference System

- MGRS uses the same zone notation as UTM coordinates. In MGRS the zone letter designator is always used and the zone is further divided into 100K meter squares. Each square is identified by a one letter easting identifier and a one letter northing identifier.
- 100K meter square easting identifiers
  - A through Z omitting I and O
  - Start at -180 degrees meridian and moving east
- 100K meter square northing identifiers
  - A through V omitting I and O
  - Start at the equator moving north
  - In the reverse for southern latitudes
- Easting and northing numbers locate position within 100K meter squares
  - Length of easting and northing numbers are identical
  - Length of numbers identifies precision, not accuracy

Color of the Universe

Red = 0.269, Green = 0.388, Blue = 0.342